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Analysis of the Transposition of the Modelling Process into Mathematical Activity at Qualifying Secondary Level

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ABSTRACT: The aim of this article is to study the teaching of the object 'modelling process' to secondary school pupils in Morocco. We are interested in the conditions and constraints of its dissemination in the mathematics classroom. The analysis is based on the Anthropological Theory of Didactics and uses the modelling process model of (Chevallard, 1999) to characterise the praxeologies (Bosch and Gascon, 2005) relating to the modelling process. Downstream of the need for knowledge relating to mathematical modelling, this study enabled us to illustrate the claim to implement didactic praxeologies specific to the modelling approach. Setting up a real situation with these pupils enabled us to identify the influence exerted by the modelling praxeologies on the pupils' results and the influence of external intervention when they were faced with a modelling situation. As a result, we noted a discrepancy between the modelling taught in the mathematics classroom and that designed by the experts.

KEY WORDS: modelling, teacher praxeologies, derivation, qualifying secondary, TAD.

1. INTRODUCTION

Educational guidelines and curricula for teaching and learning mathematics emphasise the mathematical object as a tool for modelling real or authentic situations that are problems outside mathematics. In Morocco, with the renewal of the curricula in 2006, the teaching of mathematics at secondary level has continued to use mathematical objects as a modelling tool to develop pupils' scientific culture and approach. In his work, Garcia (2005) distinguishes between two types of research: those that propose modelling as a tool to be taught and those that consider this process as a teaching object. The latter view assumes that the modelling process serves to introduce new mathematical objects, and helps to boost student motivation and contribute to and modify their attitude towards mathematics. Coulange (1998) distinguishes between two trends: teaching through modelling and teaching modelling. This new understanding of the teaching of mathematics will be developed in this work by presenting an analysis of the teaching practices of a mathematics teacher in a sub- institution of the first mathematical science bac. To study the derived concept as a modelling tool, withreference to the conditions and constraints of the diffusion of praxeologies in a mathematics class sub-institution. The a priori analysis of the mathematical praxeologies enabling the resolution of the problem that qualifying secondary school students will have to study, led us to question our theoretical framework with Orange's (2005) notion of problematisation. The presentation of the experimental conditions and the collection of our observations then led us to a detailed didactic analysis of the session observed from the point of view of mathematical knowledge. We maintain what Wozniak (2012) has defined in his work, considering that it is not a question of measuring the coherence between the preparation and the conduct of the mathematics teacher's class, but of identifying, beyond what is done, what could be, or even what should be, in order to bring to life the praxeologies of modelling.

By referring to the study by Bosch and Gascon (2005) to interpret the stages of didactic transposition of mathematical knowledge, we have adopted an epistemological point of view, independent of institutions, by defining an epistemological praxeology of reference. We used the modelling cycles of Henry (2001), Borromeo (2006) and Rodriguez (2008) to establish our modelling triangle. This theoretical framework enabled us to return to the praxeological needs of teachers to convey appropriatemodelling practices in mathematics classes.

We are attempting to answer our research question: How is the modelling process practised in the teaching of mathematics at qualifying secondary level and what are the associated praxeologies? This will enable us to study students' cognitive processes and identify the difficulties they encounter when faced with a modelling task in the mathematics classroom. We will present an analysis of the results ofthis study, carried out in class, in order to take stock of the knowledge in germ, in particular, to relate the notion of derivation among the tools for modelling optimisation phenomena.

2. MODELLING PROCESSES IN MATHEMATICS EDUCATION

2.1 Model and modelling concepts :

- **Model :**

"A model is an abstract, simplified and idealised interpretation of an object in the real world, or of asystem of relationships, or of an evolutionary process derived from a description of reality" (Henry,2001, page 151). In our analysis, the concept model is a symbolic representation in the algebraic register.

- **Modelling**

In order to develop students' modelling skills, we propose modelling activities based on the interdisciplinarity of knowledge and the school institution in the teaching of mathematics courses at secondary level. According to Chevallard (1989), the process of modelling is to determine the variables that enable us to divide up the domain of reality, then to establish relationships between thesevariables and produce mathematical knowledge.

2.2 Different modelling schemes

Thanks to the description of the reference modelling cycle in (Ruth Rodriguez 2008, p.46-4). We willlink this description to our modelling triangle.

Our triangle is made up of (SR, RS), (MR, RM), (MM, RM) and the link between the elements is atransition that will be made either by the teacher, the student or society.

2.2.1 Diagram by Henry (2001)

Henry's work (2001) also introduces a "pseudo-concrete model" between the real situation stage andthe real model stage of the modelling approach described by Chevallard.

"Identify the relevant hypotheses. This construction is guided by an initial level of knowledge of the phenomenon studied...and by the mathematical tools already mastered."

(Henry, 2001, page 145)

2.2.2 Reference Modelling Scheme (2008)

The reference modelling scheme comprises eight stages, and is closer to the modelling approach used by researchers. The author notes that in mathematics classes, most of the exercises analysed in the Bordas and Hachette textbooks were based on a real or pseudo-real model.

2.2.3 Modelling triangle at lycée :

The author Borromeo (2005) has worked on cognitive approaches for the mathematical modelling of a real situation in six stages, which are described (figure2). On the other hand, Ruth Rodriguez's scheme is made up of eight stages divided into three domains (real, pseudo-concrete, mathematical), which are described in (Figure 3) and which start from a real situation contained in an open and complex reality presented by a statement or a figure.

We have presented the various stages of the Borromeo and Ruth Rodriguez modelling cycle (Figures 2and 3), and then linked these two cycles with **our modelling triangle**, which is made up of (SR, RS), (MR, RM) and (MM, RM).

We consider that our modelling triangle is a didactic tool, which describes the entire scenario of the activity that describes an extra-mathematical phenomenon, in order to arouse the student's curiosity and interest in modelling.

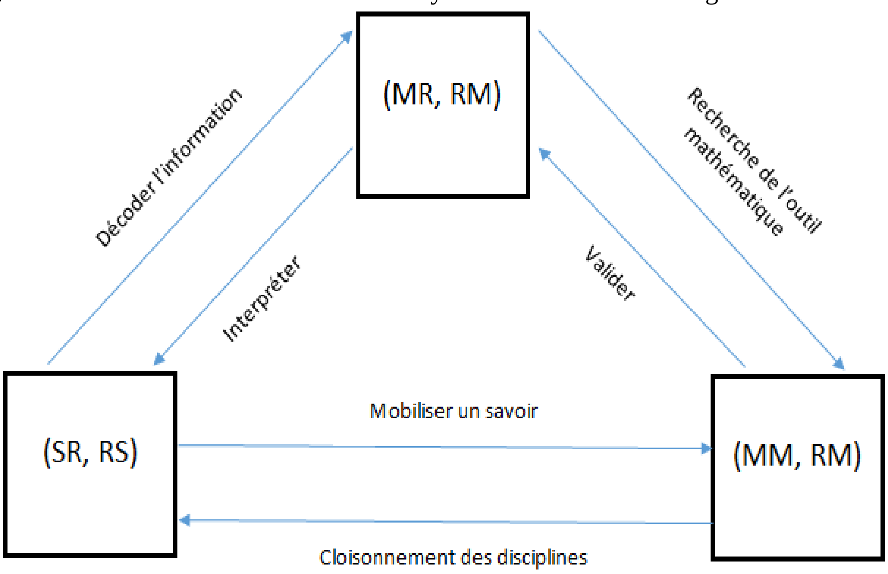

Fig. 4 modelling triangle at lycée

Couples	Task
(SR, MR)	**The student has to think about and understand the task, then decode theinformation**
(MR, MM)	**The student must look for a mathematical tool**
(MM, RM)	**The student must validate the solution purely mathematics.**
(RM, RS)	**The student has to interpret the solution, i.e.he has to go back to the problem and find out if there is a solution. he consistency of the solution.**
(SR, MM)	**The pupil must mobilise the knowledge taughtto model the problem.**
(MM, SR)	**Students must be aware of the compartmentalisation ofdisciplines**

Fig. 5 Description of the modelling triangle

SR: real situation, **MR**: real model, **MM** mathematical model, **RM** real model, **RS** real situation,
RM mathematical results.

3. ANTHROPOLOGICAL THEORY OF DIDACTICS TO CHARACTERISE PRAXEOLOGY

Anthropological Didactic Theory (ADT) considers that mathematics is taught by teachers in the mathematics classroom in Moroccan high schools. It is at the institutional level that the steering of knowledge by the teacher to the pupils is defined.

3.1 Modelling practices

In the context of solving problems that require a modelling process, we look for different techniques depending on the problem, which are classified according to Assude, Mercier and Sensevy (2007): "In the case of invisible techniques, the pupil produces a response and is in a relationship of action; in the case of weak techniques, the pupil is not only in a relationship of action (he produces a response) but also in a relationship of formulation (the technique also consists of the manipulation of standardised symbolic and linguistic tools, representations in the sense of Brousseau, 2004); in the caseof strong techniques, the student is in a relationship of action (he produces), in a relationship of formulation (he manipulates representations and also justifies this technique - he formulates a discourse on the relevance of the representations used, which are systems of notation and associated notions) and possibly in a relationship of validation (in the case where the justification becomes work on the consistency of a theory). " (Assude, Mercier, & Sensevy, 2007, p. 227).

We prefer to work on modelling praxeologies that implement either praxis or logos alone, or both at the same time.

We're now going to present the real situation to 78 students from four classes at different high schools,who will study it under the guidance of their teachers.

3.2 problematising the teaching of modelling in secondary schools

When faced with the same problematic task, i.e. one for which they have no immediate answer, two people may have constructed the problem in different ways. This construction, linked to the projects they set themselves and the knowledge they already have, will lead them to develop knowledge that will not be at all the same; whereas for an observer their solutions may seem similar" (Orange, 2005, p.73).

Again drawing on the work of Orange, who defines problematisation as "the explicit construction of a field of possibilities" (Ibid., p.70), integrating the model-building aspect into the modelling process, and as Henry defines a model: "a model is an interpretation...".

an abstract, simplified and idealised representation of an object in the real world, or of a system of relationships, or of an evolutionary process derived from a description of reality" (Henry, 2001, p.151).The notion of derivation is a modelling tool, and the modelling process is used to introduce new mathematical objects. Solving non-root problems serves to reinforce the student's motivation to contribute, changing their attitude towards mathematics, and showing the relevance of mathematics in extra-mathematical contexts. In the case of our situation, the students interviewed found it difficult to understand the context. Since they are not used to solving this kind of problem, the answer to which is not known, we are looking at how teachers teach the modelling process in the first-year mathematics science class.

Our analysis will then have to answer the question: do the students interviewed perceive that derivability is a tool for modelling a real situation?

4. METHODOLOGY

Our methodological approach to analysing the modelling process in secondary education, based on the theory of didactic transposition developed by Chevallard (1991), is that of knowledge and institutions. We will be interested in the gap between the modelling process of experts and the process activated within the mathematics classroom. The didactic transposition of the modelling process will be analysed, using derivation as a modelling tool, taking into account the mathematics syllabus in the first year of the mathematical science bac; the mathematical praxeologies in authorised textbooks; and how teachers adapt modelling to pupils.

First, we analyse the mathematical tasks and real-life situations proposed in class that ensure the introduction of the modelling process tool using the mathematical object of derivation.

Using the Anthropological Theory of Didactics of Cheval lard (1999), we analyse what has been established by the official instructions for teaching mathematics in Morocco and what is established in the two textbooks intended for students in the first year of mathematical science (L'Archipel des Maths1re année du Baccalauréat, Edition Moynier 2017 **and** the school textbook Almofid: Edition Dar Attakafa legal deposit 2006/1587 ISBNX-726-02-9981).

By identifying the different stages in the resolution of the proposed problem according to two versons, we then analyse the relevance of the components of the modelling praxeologies that are experienced during the resolution of the problem.

Given that the pupil is one of the components of the school system, we are interested in his responsesto this problem, which requires a modelling process, as well as the teacher's reaction, and in order to find out his cognitive side with regard to the modelling process, we proposed a questionnaire to secondary school mathematics teachers (see fig12).

The difficulties encountered by students in solving the problem are analysed using a modelling process.

Mathematical activity :

Version1

A citizen wants to drain the water collected at two points D and C on the roof. The front of the house has a rectangular base and part of the pipe intersects the middle of the house (see side figure). What is the position of point M so that the cost of the equipment is minimal?

Fig. 6 mathematical activity

Version 2

1) Give a suitable geometric shape.
2) Recognise the key words in the situation
3) Link the key words of the situation with at least one function.
4) Use a mathematical organisation: derivative of a real function with real variables.
5) Mathematical solution: draw up the variation table.
6) Validate and interpret.

Aim of t h e activity: As part of developing the "modelling" skill, we propose a real-life situation thatexpresses a phenomenon outside the field of mathematics.

Necessary prerequisites: real and trigonometric functions, derivation, direction ofvariation of a function and table of variation of a function.

Possible solutions: (minimise). Derive the function obtained and draw up the table of variation of thefunction.

This activity is offered to four classes (78 pupils) from different high schools.

Classes	1	2	3	4	Total
Number of students	32	14	24	8	78

Fig. 7 Breakdown of students.

During the activity, the students are asked to work individually on version 1 and, if they have anyproblems, they are given version 2.

5. A PRIORI ANALYSIS OF THIS ACTIVITY

This activity is proposed at the end of the derivation chapter to four classes of first-year mathematical science students over the two school years 2019/2020 and 2019/2018.

Solving the proposed real-life situation using modelling to determine the minimum cost of the equipment. We assume that it presents a micro-break for the pupils in relation to what is actually done in class, where there is a dominance in the types of exercises and routine problems in the classroom sub-institution. This break is due to the phenomena of the didactic contract, since the pupils have to construct the geometric figure and express the function in the algebraic register and then draw up the table of variation in the graphical register, in order to determine the minimum cost.

As the solution is not known a priori, the pupils questioned cannot validate their answers. On the other hand, the model produced by the students, with the teacher's help, provides an answer to the real situation. In this case, the modelling process validates the answer.

As our aim is to use the modelling process, the techniques we use in the sub-institution of the premier bac mathematical science class depend on the relationship between institutions and knowledge. In this real-life situation, this relationship is described by the programme on derivation (November 2007): ".... solve problems involving minimum and maximum values..." (Ministère de l'Éducation Nationale, 2007, p. 62).

In our real-life situation, we are looking for the minimum cost of the equipment, which is like lookingfor the minimum value of a function using a modelling process.

Thanks to the teacher's intervention, the pupils understand the situation, construct the geometric figure below and come up with the following hypotheses:

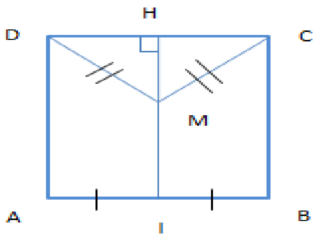

Fig. 8 Geometric figure

H1: ABCD is a rectangle.

H2: AD=h, the height of the façade. **H3**: AI=d, half the width of the façade.**H4**: DM+MC+MI=2MD+MI

Based on Henry's (2001) diagram: "Identify the relevant hypotheses. This construction is guided byan initial level of knowledge of the phenomenon studied... and by the mathematical tools already mastered". (Henry, 2001, p145).

- At this stage, the students have chosen x=MH, a "pseudo-concrete model (PCM)".

This MPC is also introduced by Henry (2001) between the real situation and the real model of the modelling approach, Chevillard (1999).

- After considering the variable x, the teachers ask the students surveyed to

determine $l(x)$ the length of the tube as a function of x, i.e. determine the mathematical model: "in this stage, we establish a set of equations or mathematical formulations that represent the properties of the model and the assumptions made. The subjects mainly use external representations, such as formulas or graphs. The subjects' verbal statements belong to a mathematical level and make less reference to reality". (Ruth Rodriguez, 2008, p.47)

$$l(x) = \sqrt{x^2 + d^2} + h - x$$

- Mathematical solution by the students in the following stages:

The techniques used by the students are T1 derivation and T2 the variation table. To determine theminimum value o f $l(x)$.

$$l'(x) = \frac{2x-\sqrt{x^2+d^2}}{\sqrt{x^2+d^2}}$$

- T1: By syntactically checking the use of derivation rules
- T2: variation table

- When solving the proposed problem, the student determined the function and then used derivation as a mathematical application to model the real situation. This develops the student'screative spirit and extra-mathematical problem-solving skills.
- Teaching the modelling process shows students the relevance of mathematics to problems in other disciplines (physics, economics, etc.).
- By solving the proposed problem, the student establishes an important representation of mathematics as a field of life problems.

Simulation: https://drive.google.com/file/d/1bLqrf65Pxc8pneFSgdzRAvMBSjFwLoO_/view

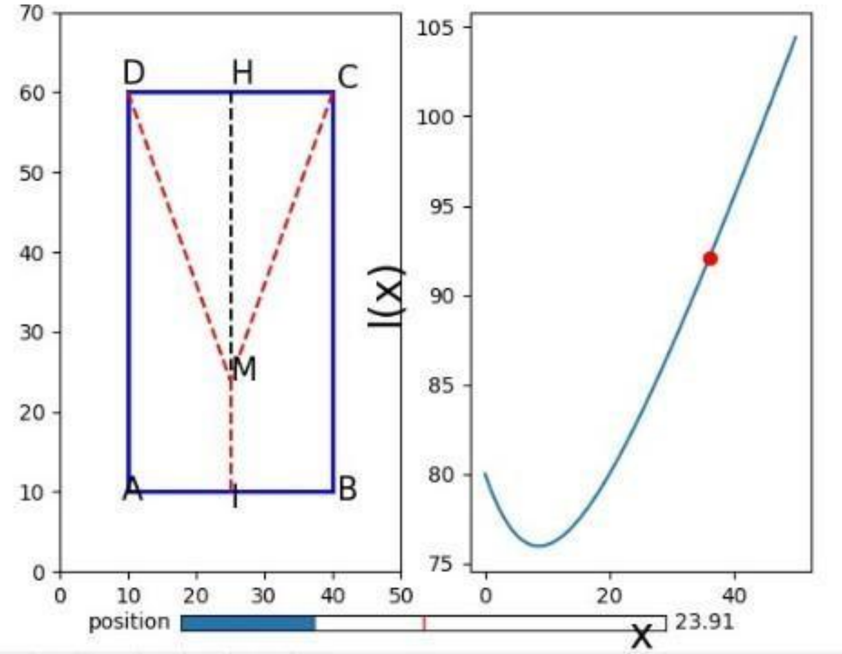

Fig. 9 simulation

For **version 1**, we gave no indication and left it up to the students to answer individually. We found that two of the 78 students who managed to answer said that they were going to look for the minimum of a function.
Teacher: Have you ever encountered this kind of problem? Pupil : No,this is the first time.
Professor: why did you answer, we're looking for the minimum of a function?

Pupil: I inspired the answer in the derivatives and extremums paragraph.
However, the students questioned were unable to determine the function, which is why we proposed **version2** ? so that the students could take part in the resolution process, i.e. they could specify the choice of framework and representation register with fewer intermediate questions.

Stage 1: a cognitive process
Q1, construct the geometric figure.
Q2, identify the key words: decode the information.

Classes	1	2	3	4	Total	%
Q1 : RC	2	1	1	2	6	7,69%
Q2 : RC	2	1	1	1	5	6 ,64%
%	6,25%	7,14%	4,16%	25%		

Fig.9: Distribution of students' correct answers (CR).

Stage 2: mathematical phase.

Q3, determine the length of the channel: determine the function.

In Q4, calculate the derivation of the function and draw up the table of variation.

Classes	1	2	3	4	T	%
Q1 : RC	20	12	18	2	52	66 ,66%
Q2 : RC	25	13	20	6	61	78,20%

Fig.10: Distribution of students' correct answers

Stage 3: reality phase.

Q5, validate mathematical results (mathematical tasks). Q6,communicate the results (interpret the results).

Most of the students interviewed found it difficult and said that they had never encountered thistype of non-rotating question.

Analysis and report on these workshops:

For stage 1: the inadequate results (6.7% success rate) show that the pupils are finding it difficult to understand the task required and decode the information in the real situation. The questions in this phaseare non-rotating for the pupils and require a cognitive process.

To ensure the transition from the real situation to the real model, the teacher must intervene to helpthe students answer the question "What is it about?

For stage 2: In view of the interesting results (78.8% success rate), the students were able to determine the length of the canal as a real function and use the derivation they had acquired to carry out a task and a technique, using a technology justified by a theory.

Task	Technical	Technology	Theory
Looking for the minimum	Primary derivation	Transactions on derivatives	Function space
	Search for a function		

(Fig.11)

Frame and register :

Representation	Frame	Register
The real situation	Analysis	Statement of changes

(Fig.12)

For stage 3: the students are unable to interpret the results, as they are not used to communicating resultsthat describe reality.

Questionnaires on modelling in the mathematics classroom for 20 secondary schoolteachers :

Questions	RO	RN
Define word modelling	17	3
Have you had any training in modelling?	5	15
Do you provide students with real-life situations thatrequire modelling?	17	3
Do you teach modelling as a subject at lycée?	4	16
Do you teach modelling as a tool at lycée?	12	8
Is the teaching of modelling a loss oftime?	2	18
Do students find it difficult to model?	19	1
Is modelling part of the mathematics curriculum at lycée?	8	12
Do the official instructions require teachers toteaching modelling?	8	12
Is there a gap between the modelling done by scholars and themodelling done in high school?	18	2
Are the exercises provided in school textbooks sufficient fordevelop modelling skills?	1	19

(**Fig. 13**)

RO: yes answer, **RN**: no answer.

Analysis of results :

20% of teachers replied that modelling is an object.

40% of teachers replied that modelling is part of the curriculum and required by the officialinstructions.

95% of teachers said that there is a gap between the modelling done by scholars and themodelling done at lycée.

95% said that the exercises proposed were not sufficient to develop modelling skills.

Analysis of manuals :

The lesson: derivation for 1bac science mathematics students A.

The ALMOFID textbook: Edition DAR Attakafa legal deposit 2006/1587 ISBNX-726-02-9981

Lessons	Number ofexercises	Number of exercises to be modelled	Number of exercises to be modelledresolved
Bypass	104	11	1
The functions	57	0	1

(Fig.14)

Solving problems involving minimum and maximum values accounts for 12.5% of the lesson and 10.57% of the exercises proposed require a modelling process, and the majority of teachers do not propose problems requiring A modelling process. Secondary school students do not benefit from training in the modellingprocess.

Derivation is a concept for modelling an extra-mathematical situation from other fields (physics, chemistry, life and earth sciences, economics, etc.). Thanks to the praxeological analysis, the only techniques are the search for the function and the derivation carried out by the pupil, then he draws up the table of variation.

The Archipelago of Maths textbook:

Solving problems involving minimum and maximum values accounts for 12.5% of the lesson, and 14.05% of the exercises proposed require a modelling process.

Lessons	Number ofexercises	Number of exercises to be modelled	Number of exercises to be modelledresolved
Bypass	57	8	1
The functions	46	4	0

(Fig. 15)

6. OUTLOOK AND CONCLUSIONS

Following the praxeological analysis of the exercises proposed in the two manuals in the sub- institutional mathematics class of the first mathematical science in Morocco, the modelling process is reduced to the mathematical domain, to determining the minimum using the mathematical organisationderivation of the function to be determined. On the other hand, the only elements of praxeology experienced during modelling in the high school are: the techniques for determining the function and the use of the mathematical organisation derivation. This explains the absence of the other elements and the fact that the modelling approach in the mathematics classroom sub-institution differs from that of the experts, and that the situation statement represents the pseudo-concrete model as described by Henry (2001). We will be looking for other mathematical organisations to model situations from other disciplines. And to encourage interaction between teachers from other sub-institutions.

We found that first-year bac students had difficulty with the technique of determining the function and drawing up the table of variation after calculating the first derivative. And that they have difficulties with the derivation as a modelling tool. We will try to find out what difficulties students encounterwhen learning to teach the mathematical object derivation.

We analyse the intervention of teachers who help students during the transition between the real situation and the real model, in order to develop the modelling skill in secondary education we try to encourage the relationship between the elements of the triplet (real situation, teacher, student). On the other hand, for future studies we are trying to work on other mathematical objects to see if we can see the same results in other activities involving extra-mathematical fields that use modelling as an object.

To characterise the modelling approach, we used the theory of didactic transposition (chevallard 1991)to establish the analysis of the didactic transposition of modelling in secondary education from the official instructions, the authorised manuals based on the praxeology in the mathematics class sub- institutions and how the teachers teach the modelling process.

In our analysis of the transposition of the modelling process in first-year mathematics textbooks, students are asked to determine the min or max, sometimes they are given the function, and in most cases it is up to them to determine the function and use the derivation.

The determination of the function is almost absent in the problems proposed in the two mathematical manuals which require modelling and, above all, these problems are applications of the chapter (derivation), i.e. the model is already provided.

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