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Novel Extended One Parameter of 10 Mixture Gamma (NEG10) Distributions and Application

Nurul Ulumi[1], Rado Yendra[2], Rahmadeni[3], Arir Pani Desvina[4], Muhammad Marizal[5]
[1,2,3,4,5] Universitas Islam Negeri Sultan Syarif Kasim Riau

Corresponding Author: Nurul Ulumi

ABSTRACT: A new one parameter lifetime distribution named, 'the novel extended 10-mixture gamma (NEG10) distribution' for modeling lifetime data from engineering, has been proposed. The method of maximum likelihood has been discussed for estimating its parameter. The goodness of fit of the proposed distribution over one parameter Lindley, Sujatha, Amarendra, Devya, Shambhu, mixture 7 component gamma distribution, mixture 8 component gamma distribution and mixture 9 component gamma, and NEG10 distributions have been given with two real lifetime data sets. The maximum likelihood method will be employed to estimate the parameter values of the distributions used in this study. Additionally, graphical assessments (density-density plot) and numerical criteria (Akaike's Information Criterion (AIC), -2*log likelihood) will be utilized to determine the best-fitting model. In most instances, the results obtained from graphical assessments were consistent but differed from the numerical criteria. The model with the lowest values of AIC and -2*log likelihood was selected as the best fit. Overall, the NEG10 distribution was identified as the most suitable model.

KEY WORDS: Linley, Sujatha, Amarendra, Devya, Shambhu, Mixture gamma distribution.

INTRODUCTION

Recently, a number of one parameter lifetime distributions have been introduced by Shanker namely Sujatha, Amarendra, Devya and Shambhu. Shanker [2] introduced continuous distribution named, Sujatha distribution defined by its probability density function (pdf)

$$f(x) = \frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-x}, \quad x>0$$

This distribution is a three-component mixture of a gamma (1, θ) distribution, a gamma (2, θ) distribution and a gamma (3, θ) distribution with their mixing proportions $\frac{\theta^2}{\theta^2+\theta+1}$, $\frac{\theta}{\theta^2+\theta+1}$ and $\frac{1}{\theta^2+\theta+1}$ respectively. Shanker [3] introduced one parameter lifetime distribution named Amarendra distribution defined by probability density function (pdf) as

$$f(x) = \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}(1+x+x^2+x^3)e^{-x}, \quad x>0$$

Amarendra distribution is a mixture of a gamma (1, θ) distribution, a gamma (2,θ) distribution , a gamma (3,θ) distribution and a gamma (4,θ) distribution with their mixing proportions $\frac{\theta^3}{\theta^3+\theta^2+2\theta+6}$, $\frac{\theta^2}{\theta^3+\theta^2+2\theta+6}$, $\frac{2\theta}{\theta^3+\theta^2+2\theta+6}$ and $\frac{6}{\theta^3+\theta^2+2\theta+6}$ respectively. Shanker [4] proposed another one parameter lifetime distribution named Devya Distribution, the probability density function can be obtained as

$$f(x) = \frac{\theta^5}{\theta^4+\theta^3+2\theta^2+6\theta+24}(1+x+x^2+x^3+x^4)e^{-x}, \quad x>0$$

Devya distribution is a mixture of a gamma (1, θ) distribution, a gamma (2,θ) distribution , a gamma (3,θ) distribution, a gamma (4,θ) distribution and a gamma (5,θ) distribution with their mixing proportions $\frac{\theta^4}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{\theta^3}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{2\theta^2}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, $\frac{6\theta}{\theta^4+\theta^3+2\theta^2+6\theta+24}$, and $\frac{24}{\theta^4+\theta^3+2\theta^2+6\theta+24}$ respectively. Shanker [5] has also obtained a shambhu distribution, the probability density function can be obtained as

$$f(x) = \frac{\theta^6}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}(1+x+x^2+x^3+x^4+x^5)e^{-x}, \quad x>0$$

Shambhu distribution is a mixture of a gamma (1, θ) distribution, a gamma (2,θ) distribution , a gamma (3,θ) distribution, a gamma (4,θ) distribution, a gamma (5,θ), and a gamma (6,θ) distribution with their mixing proportions $\frac{\theta^5}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$, $\frac{\theta^4}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$, $\frac{2\theta^3}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$, $\frac{6\theta^2}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$, $\frac{24\theta}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$ and $\frac{120}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}$ respectively. Although these lifetime distributions give better fit than the classical exponential and Lindley distributions [1], there are still some lifetime data where these distributions are not suitable due to their theoretical and applied point of view. In this study, we investigate a brand-new one-parameter distribution as an extension of other distributions, such as Lindley distribution, Sujatha, Amarendra, Devya, and Shambuu distributions. Using the addition of some gamma distributions such as gamma (7, θ), gamma (8, θ) and gamma (9, θ) with a specified proportion, the produced distribution is called the novel extended 10-mixture gamma (NEG10) distribution. Usefulness in producing symmetric and asymmetric probability density functions are perfect choices for lifetime phenomena modeling. The methods of maximum likelihood estimation are discussed to estimate its parameters. The goodness-of-fit of the proposed distribution over a one-parameter, Lindley, Sujatha, Amarendra, Devya, Shambhu, and NEG10 distributions was determined using real lifetime data sets.

The novel extended one parameter 10-mixture gamma distribution (NEG10)

In search for a new lifetime distribution, we have proposed a new lifetime distribution which is better than Lindley, Sujatha, Amarendra, Devya, and Shambhu distributions for modeling lifetime data by considering a ten-component mixture of a gamma (1, θ) distribution, a gamma (2,θ) distribution , a gamma (3,θ) distribution, a gamma (4,θ) distribution, a gamma (5,θ) distribution, gamma (6, θ) distribution, a gamma (7,θ) distribution , a gamma (8,θ) distribution, a gamma (9,θ) distribution and a gamma (10,θ), with their mixing proportions

$$p_1 = \frac{\Gamma(1)\theta^9}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_2 = \frac{\Gamma(2)\theta^8}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_3 = \frac{\Gamma(3)\theta^7}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_4 = \frac{\Gamma(4)\theta^6}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_5 = \frac{\Gamma(5)\theta^5}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_6 = \frac{\Gamma(6)\theta^4}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_7 = \frac{\Gamma(7)\theta^3}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_8 = \frac{\Gamma(8)\theta^2}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_9 = \frac{\Gamma(9)\theta}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

$$p_{10} = \frac{\Gamma(10)}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}$$

where $\Gamma(\alpha) = (\alpha - 1)!$, α = positif integer. The new density function can be generated based on the mixed theory of the Probability density function

$f(x) = \sum_{i=1}^{k} p_i f_i(x)$ where $f_i(x) = gamma(i,\theta), p_i, i = 1,2,\cdots,10$ and $\sum_{i=1}^{k} p_i = 1$. The new Probability density function can be generated as follows

$$f(x) = \frac{\theta^{10}}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)} (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9)\, e^{-x\theta},\ x > 0, \theta > 0$$

We would call this distribution, the novel extended 10-mixture gamma distribution (NEG10)

Parameter Estimation

Parameter estimation is the most important part of modeling probability. The maximum likelihood method will be used in estimating the parameter. The likelihood and log likelihood function as the most important step to be able to obtain the parameter value of a NEG10 distribution can be given as follows, respectively

$$L(\theta;x) = \left(\frac{\theta^{10}}{\Gamma(1)\theta^9+\Gamma(2)\theta^8+\Gamma(3)\theta^7+\Gamma(4)\theta^6+\Gamma(5)\theta^5+\Gamma(6)\theta^4+\Gamma(7)\theta^3+\Gamma(8)\theta^2+\Gamma(9)\theta+\Gamma(10)}\right)^n \prod_{i=1}^{n}(1 + x_i + {x_i}^2 + {x_i}^3 + {x_i}^4 + {x_i}^5 + {x_i}^6 + {x_i}^7 + {x_i}^8 + {x_i}^9)\ e^{-\theta \sum_{i=i}^{n} x_i}$$

$$l(\theta;x) = 10\,n\,log(\theta) - nlog\big(\Gamma(1)\theta^9 + \Gamma(2)\theta^8 + \Gamma(3)\theta^7 + \Gamma(4)\theta^6 + \Gamma(5)\theta^5 + \Gamma(6)\theta^4 + \Gamma(7)\theta^3 + \Gamma(8)\theta^2 + \Gamma(9)\theta + \Gamma(10)\big) + \sum_{i=1}^{n} log(1 + x_i + {x_i}^2 + {x_i}^3 + {x_i}^4 + {x_i}^5 + {x_i}^6 + {x_i}^7 + {x_i}^8 + {x_i}^9) - \theta \sum_{i=1}^{n} x_i$$

Parameter estmation can be done by performing a homogeneous solution of the non-linear function produced by the derivative of the log likelihood function to θ, $\frac{dl(\theta;x)}{d\theta} = 0$. This solution can be done using one of the numerical methods, namely Newton-Rhapson. The selection of the best model (Goodness of Fit Test) will be carried out using the graph method (density-density plot) and the numerical method (-2 * Log likelihood and AIC). AIC or Akaike Information Criterion can be searched using the following formula

AIC = -2 * Log likelihood + 2 * p (p is the number of parameters).

Goodness of Fit and Applications

In this section, the goodness of fit and applications of NEG10 distribution to real data sets using maximum likelihood estimate has been presented and the fit has been compared with one parameter Lindley, Sujatha, Amarendra, Devya, Shambhu, mixture 7 component gamma distribution, mixture 8 component gamma distribution, mixture 9 component gamma distribution with each probability density function attached in table 1.

Table 1. Probability Desity Function

	f (x)
Lindley	$\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}$
Sujatha	$\frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x}$
Amarendra	$\frac{\theta^4}{\theta^3+\theta^2+2\theta+6}(1+x+x^2+x^3)e^{-\theta x}$
Devya	$\frac{\theta^5}{\theta^4+\theta^3+2\theta^2+6\theta+24}(1+x+x^2+x^3+x^4)e^{-\theta x}$
Shambhu	$\frac{\theta^6}{\theta^5+\theta^4+2\theta^3+6\theta^2+24\theta+120}(1+x+x^2+x^3+x^4+x^5)e^{-\theta x}$
k = 7	$\frac{\theta^7}{\theta^6+\theta^5+2\theta^4+6\theta^3+24\theta^2+120\theta+720}(1+x+x^2+x^3+x^4+x^5+x^6)e^{-\theta x}$
k=8	$\frac{\theta^8}{\theta^7+\theta^6+2\theta^5+6\theta^4+24\theta^3+120\theta^2+720\theta+5040}(1+x+x^2+x^3+x^4+x^5+x^6+x^7)e^{-\theta x}$
k = 9	$\frac{\theta^9}{\theta^8+\theta^7+2\theta^6+6\theta^5+24\theta^4+120\theta^3+720\theta^2+5040\theta+40320}(1+x+x^2+x^3+x^4+x^5+x^6+x^7+ +x^8)e^{-\theta x}$

k is number of component gamma distribution

The following real lifetime data-sets from engineering has been considered.

Data set: The data set is the strength data of glass of the aircraft window reported by Fuller *et al* (1994)[6]: 18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381.

In order to compare the goodness of fit of these distributions,, AIC (Akaike Information Criterion), and -2*Log Likelihood for real data sets have been computed and presented in table 2. The formula for computing AIC is as follows :

AIC = -2 * Log likelihood + 2 * p (p is the number of parameters).

Table 2. Parameter, AIC and -2*log likelihood for Dataset

	θ	AIC	-2*Log Likelihood
Lindley	0.0629	255.9884	253.9884
Sujata	0.0956	243.5031	241.5031
Amarendra	0.1282	235.4087	233.4087
Devya	0.1608	229.6854	227.6854
Shambhu	0.1933	225.3981	223.3981
k = 7	0.2258	222.0736	220.0736
k = 8	0.2583	219.4411	217.4411
k = 9	0.2908	217.3308	215.3308
k = 10	0.3233	**215.6289**	**213.6289**

The model with the lowest values of AIC and -2*Log Likelihood was selected as the best model. It is obvious from above table that NEG10 distribution gives much closer fit than Lindley, Sujatha, Amarendra, Devya, Shambhu, mixture 7 component gamma distribution, mixture 8 component gamma distribution and mixture 9 component gamma distribution and hence it may be preferred to Lindley, Sujatha, Amarendra, Devya, Shambhu, mixture 7 component gamma distribution, mixture 8 component gamma distribution and mixture 9 component gamma distribution for modeling various lifetime data.

The fitted density-density plots of considered distributions (Lindley, Sujatha, Amarendra, Devya and Shambhu) for the given datasets have been presented in Figure 2, while figure 3 is the fitted density-density plots of considered distributions (mixture 7 component gamma distribution, mixture 8 component gamma distribution and mixture 9 component gamma, and NEG10) and Comparison of AIC values for each model is also shown in Figure 3, where k is the number of gamma distribution mixtures. The goodness of fit in Table 2 and the fitted plots of distributions for the dataset in figure 2 and figure 3 show that NEG10 distribution provides best fit over Lindley, Sujatha, Amarendra, Devya, Shambhu, mixture 7 component gamma distribution, mixture 8 component gamma distribution and mixture 9 component gamma distribution and therefore NEG10 distribution can be considered as the most suitable lifetime distribution for modeling lifetime data from engineering. Furthermore, based on Figure 1, it can be concluded that in this study, increasing the number of distribution mixtures will result in a better model.

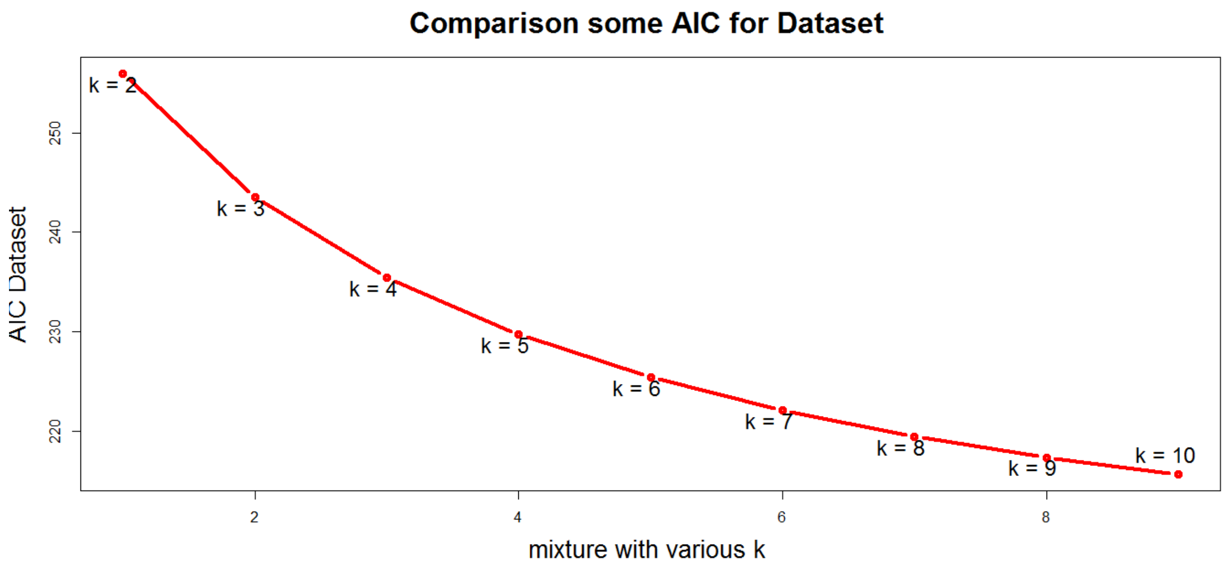

Figure 1. The Comparison AIC value for various model with different k (number of mixture)

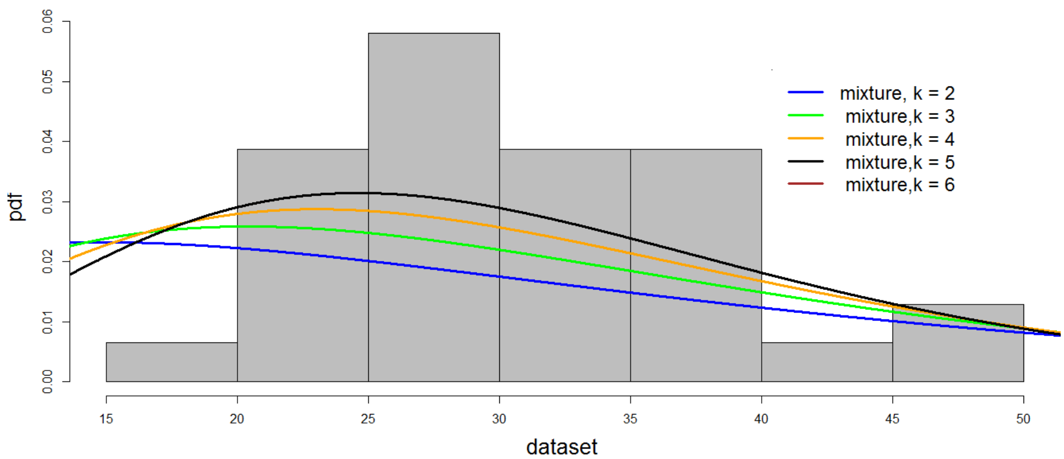

Figure 2. fitted probability density function (pdf) of Lindley (k=2), Sujatha (k=3), Amarendra (k=4), Devya (k=5), and Shambhu (k=6),

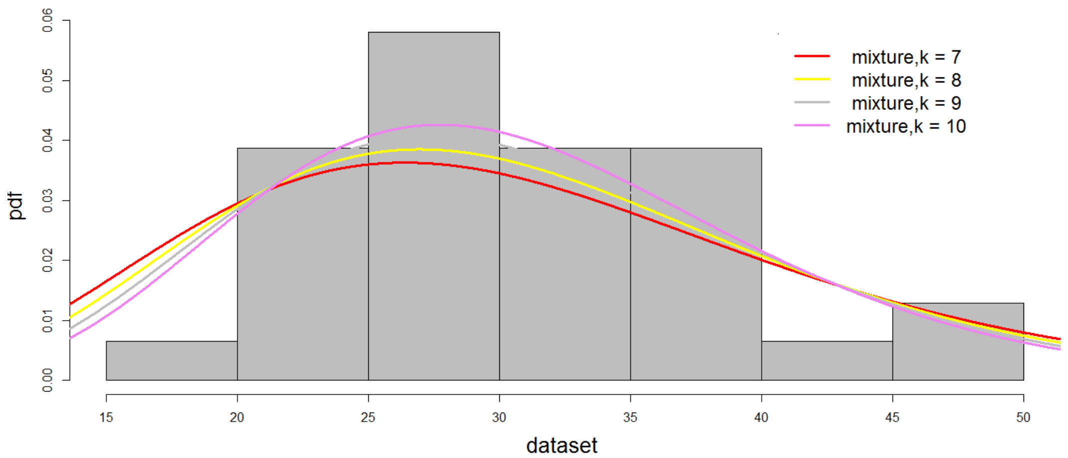

Figure 3. fitted probability density function (pdf) of mixture 7 component gamma, mixture 8 component gamma, mixture 9 component gamma, and mixture 10 component gamma distribution (NEG10).

CONCLUSION

In this study, the focus is to compare a new probability model, namely NEG10, with other 1-parameter probability models, such as Lindley, Sujatha, Amarendra, Devya, and Shambhu. The NEG10 probability model formed from a mixture of 10 gamma distributions with certain weights was found to be more accurate than the other models considered in this study. Using the same dataset, we conclude that increasing the number of gamma-distributed mixtures can reduce the AIC value.

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